

11-2 Arithmetic Series

Series is the sum of a sequence

8, 12, 16, 20, 24, sequence

$8 + 12 + 16 + 20 + 24$ series

$$S_1 = 8$$

$$S_2 = 8 + 12 = 20$$

$$S_3 = 8 + 12 + 16 = 36$$

$$S_4 = 56$$

$$S_5 = 80$$

In general, $a_1, a_2, a_3, a_4, \dots, a_n$

$$S_n = a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

Σ sigma summation sign

Σ

$$8 + 12 + 16 + 20 + 24$$

Find a_n .

$$a_n = a_1 + (n-1)d$$

$$= 8 + (n-1)4$$

$$= 8 + 4n - 4$$

$$a_n = (4n + 4)$$

$$\sum_{n=1}^5 (4n + 4)$$

$$\sum_{n=1}^5 (4n + 4)$$

"the sum of $4n + 4$ as n goes from 1 to 5"

$$\text{Expanded form} = 8 + 12 + 16 + 20 + 24$$

$$\sum_{n=3}^7 (2n - 3)$$

Put in Expanded form.

$$= 3 + 5 + 7 + 9 + 11$$

Put the following series into sigma notation:

$$3 + 6 + 9 + \dots + 36$$

$$\sum_{n=1}^{12} (3n)$$

$$a_n = 3 + (n-1)3$$

$$a_n = 3n$$

$$36 = 3n$$

$$12 = n$$

Challenge:

Find the sum of the integers from 1 to 100

$$\begin{array}{rcl} 1 + 100 & = & 101 \\ 2 + 99 & = & 101 \\ 3 & 98 & 101 \end{array} \times 50$$

$$\frac{n}{2}(a_1 + a_n) \quad 5050$$

Friedrich Gauss

$$a_1 =$$

$$a_{100} =$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex:

Find the sum of the given sequence:

5, 10, 15, 20

$$S_4 = \frac{4}{2}(5 + 20)$$

$$S_4 = 50$$

Ex:

Find the sum of the given sequence:

3, 6, 9, 12, 15, 18

Ex:

Find the sum of the first 50 terms of the given sequence:

3, 6, 9, 12, 15, 18, ...

$$a_{50} = 3 + (50-1)3$$

$$= 150$$

$$S_{50} = \frac{50}{2}(3 + 150)$$

$$= 3825$$

When you don't have the last term, either find it or use this formula.

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$a_1 + a_n$$

$$S_{50} = \frac{50}{2}(2(3) + 49 \cdot 3)$$

$$= 3825$$

Ex:

$$a_n = -14$$

$$n = 9$$

$$d = -8$$

$$S_9 = \underline{162}$$

$$a_n = a_1 + (n-1)d$$

$$-14 = a_1 + 8(-8)$$

$$50 = a_1$$

$$S_9 = \frac{9}{2}(50 + -14)$$

HW p586

15-37 odd, 40